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A PRACTICAL APPROACH TO ANALOG COMPUTERS

by

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Abstract: The purpose of this paper is to introduce engineers to the basic principles of analog computation. It explains briefly how this versatile problem-solving technique helps to increase engineering efficiency. Several types of computing modules from presently available General Purpose Analog Computers are illustrated and described. These modules represent typical details of hardware.

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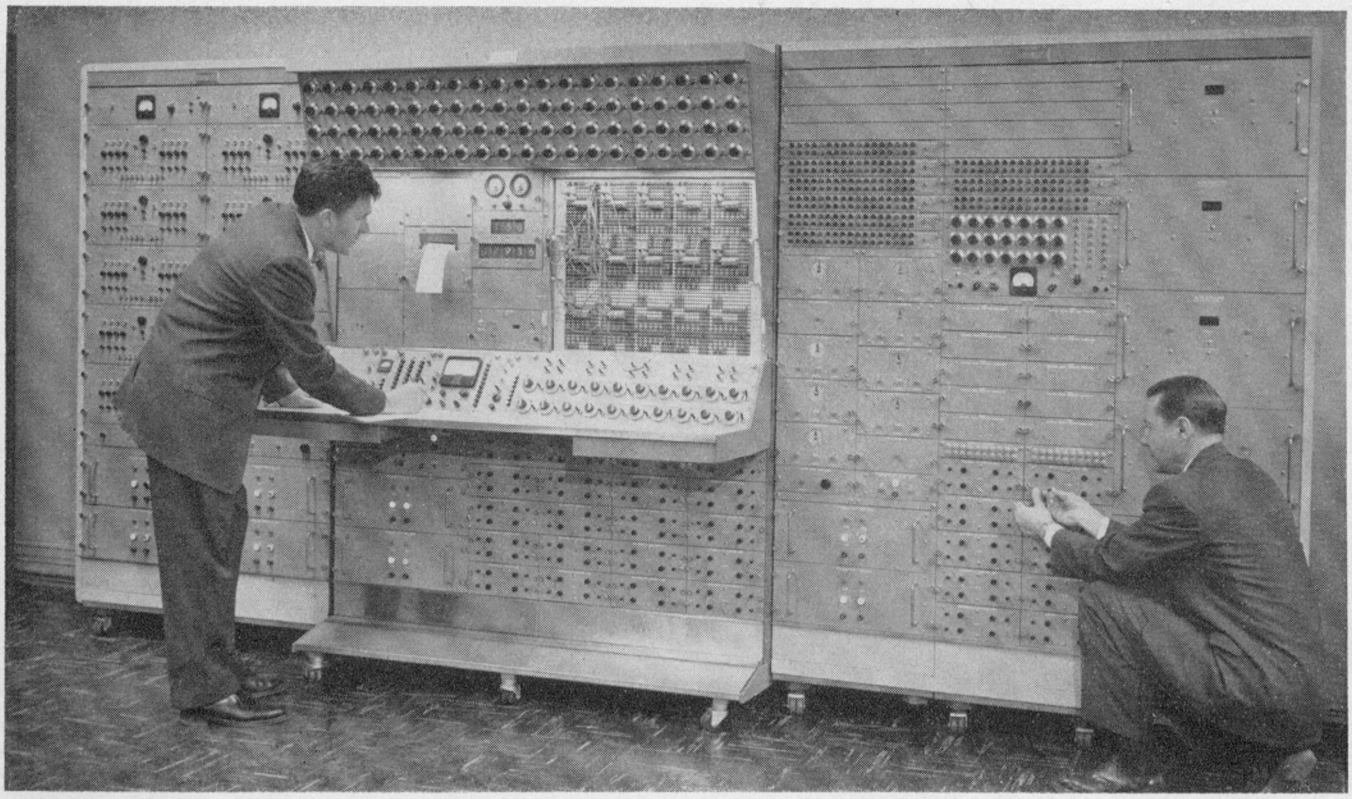

FIG. 1. PACE 231R analog computer.

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The analog computer permits any engineer to analyze and synthesize a system with speed and efficiency. Here is a practical introduction to the analog computer, and a guide to its use.

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TODAY'S ANALOG COMPUTER has a substantial record of achievement, especially in solving problems of guided-missile trajectories and airframe design. Its use is spreading rapidly into other fields, including process simulation, analysis of control systems, power plant design, etc.

At first sight, the numerous dials, switches, and indicators (Fig. 1) make the analog computer appear complex. However, the computer consists of a large number of a few basic types of similar building blocks. This is why the average engineer can be taught to operate it in as little as 8 hours; with a week of practice and study he can successfully set up and run worth-while problems. This article has been written for such an engineer—one who has heard of what analog computers can do, and who wants to know how the computer would help to solve his engineering problems and thus increase his engineering efficiency.

Although several types of analog computers are available, the basic principles of all of them are similar, so that a description of one type will provide an adequate introduction to the field. Reference will be made to the EAI Model 231R electronic computer as a typical general-purpose analog computer (Fig. 1) to illustrate details of hardware.

The Electric Analog of a Physical Variable

On an analog computer, physical variables such as weight, temperature, or area are represented by voltages—that is, voltage is the electrical analog of

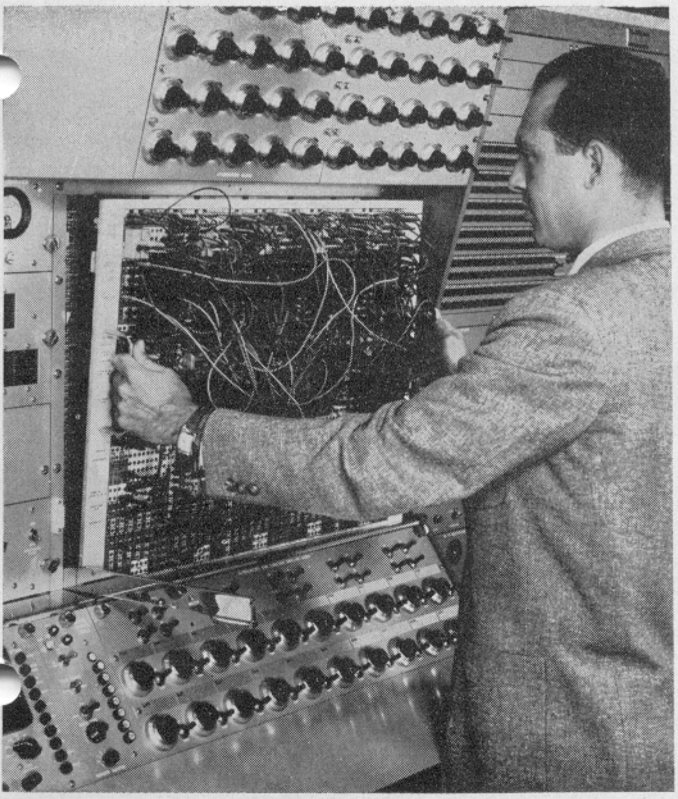


FIG. 2. PRE-PATCHED panel being inserted into computer.

the variable being analyzed, which can be mechanical, hydraulic, pneumatic or even electrical in nature. Arbitrary *scale factors* relate voltages in the computer to the variables in the problem being solved.

The computer components are designed to operate within an output voltage range of ± 100 volts, and all computer variables are scaled to lie within this range. Thus, a temperature T which can vary from 0 to 1000°C is represented on the computer by a voltage that varies from 0 to +100 V (100 V=1000°C; 1°C= 0.1 V). The scale factor would be 1/10 volt per degree Centigrade.

As another example, a displacement x which varies from −5 to +10 inches could be represented by a voltage that varies from −50 to +100 V, and the scale factor would be 10 volts per inch.

The computer components are interconnected so that the voltages in the computer are related by the same mathematical equations as the original physical variables. Thus, if a voltage on the computer represents temperature in a chemical reactor, and is scaled for 1 volt per 5 degrees Centigrade, a graph showing this voltage increasing from 10 to 100 volts would mean that the reactor temperature varied from 50 to 500 degrees. It is from this analogy between the problem variable and the computer voltage that the *analog computer* derives its name.

Computer Elements

The computer consists of a few basic components, or "building blocks," which perform mathematical operations such as addition, subtraction, multiplication, division, integration, etc. The basic components are (1) amplifiers, (2) potentiometers, (3) multipliers and (4) function generators. These components can be connected to solve a variety of equations.

In practice, the computer elements are connected, or "patched," to set up (and solve) the equations which are derived from the dynamics of the system to be analyzed. The interconnections are made by means of a *patch panel* (Fig. 2). Every computing component has its input and output terminations on this panel, and leads can be run between any two holes on the panel, connecting the components in any desired manner. The panel is removable, and problems are usually patched external to the computer. Hence the machine is not tied up while the connections are being made; problems can be patched on individual patch panels while the computer is solving another problem.

The Amplifier

The basic computer component is the *amplifier*, a direct-coupled amplifier with d-c gain of 100 million or greater. This means that an input voltage of 10^{-8} volt (only 0.01 microvolt) could produce an output of a volt or more. There is also a sign (phase) change associated with the amplifier—that is, a positive voltage input produces a negative voltage output, and *vice-versa.* The symbol for an amplifier is shown in Fig. 3.

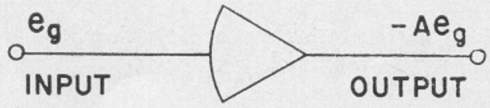


FIG. 3. HIGH-GAIN amplifier. The input voltage is multiplied by gain A (10^8 or greater); minus sign indicates phase reversal in amplifier.

The Summing Amplifier

The amplifier can be used to *add* voltages as shown in Fig. 4. R_1 and R_2 are called *input* resistors, and

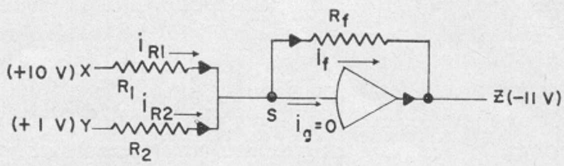


FIG. 4. HIGH-GAIN amplifier used as a summer.

R_f is the *feedback* resistor. Applying Kirchoff's law for current at the summing junction, we have:

$$i_{R1} + i_{R2} + i_{Rf} + i_g = 0$$

where i_g is the input current to the amplifier. If the voltage at point S is e_s, applying Ohm's law:

$$\frac{X - e_s}{R_1} + \frac{Y - e_s}{R_2} + \frac{Z - e_s}{R_f} = -i_g \quad (1)$$

The amplifier itself is designed to draw as little grid current (i_g) as possible from the inputs X and Y. As i_g is negligible in comparison with the other currents ($i_g \cong 10^{-9}$ amp), the right-hand side of equation (1) can be set equal to zero without significant error. Since the amplifier output $Z = Ae_s$, where A = the gain of the amplifier, $e_s = -Z/A$. Substituting $-Z/A$ for e_s in equation (1) and solving for the output voltage Z:

$$Z = -\left[\frac{X}{R_1} + \frac{Y}{R_2}\right] \bigg/ \left[\frac{1}{R_f} + \frac{1}{A}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_f}\right)\right] \quad (2)$$

Now we see the reason for the extremely large gain, A. If A is sufficiently large, we may ignore the terms involving 1/A, and obtain the approximations:

$$Z = -\left[\frac{R_f}{R_1}(X) + \frac{R_f}{R_2}(Y)\right] \quad (3)$$

If all three resistors are equal, then $Z = -(X + Y)$. Except for the sign reversal, we have succeeded in adding two voltages. If all resistors are equal, the error introduced by ignoring the 1/A terms is less than 3 parts in 100 million.

This method is not limited to two inputs. With additional input resistors, three or more voltages can be added.

Multiplying By a Constant

When the input and feedback resistors are equal, the amplifier is a simple summer. However, when the input resistor is a fraction of the feedback resistor, the output (Z) equals the input (X) multiplied by the factor (R_f/R_1), as shown in Equation 3.

Hence, if the feedback resistor R_f is 10 times the input resistor, the output voltage is 10 times the input voltage.

In actual computer diagrams the resistors are not drawn, but a *gain* of 1 or 10 is indicated, as shown in Fig. 5.

With a single input (gain of 1) the amplifier is simply an inverter (Fig. 6). With summers and inverters, subtraction can be performed as shown in Fig. 7. The amplifier at left is simply an inverter; the amplifier at right is summing its two inputs.

The Potentiometer

We now can add and subtract voltages, change sign (multiply by —1) and multiply by 10. To multiply a voltage by a factor other than 1 or 10, a *potentiometer* ("pot" for short) is used. Fig. 8 shows a pot schematic; Fig. 9 shows the symbol for a pot (a circle); Fig. 10 shows the appearance of pots on a computer; Fig. 11 shows use of pots.

In Fig. 8, the voltage on the arm of the potentiometer is k times the input voltage, where the factor k can be set to any value between zero and one. Potentiometers on the computer can be equipped with a calibrated dial, and the value of k set directly on the dial—if high accuracy is not required.

However, this setting can be in error for several reasons—electrical loading, mechanical misalignments of the dial, backlash, etc. To avoid these errors and to set a pot very accurately, the potentiometer is set by monitoring its output with the actual load connected to the pot wiper. Each pot is equipped with a switch which, when depressed, disconnects the input terminal of the pot from the circuit and connects it to a fixed voltage (+100 volts). The arm of the potentiometer is monitored on a voltmeter and the dial is rotated until the correct reading is obtained. For example, to set a pot to 0.5317, the operator depresses the switch and then rotates the dial until 53.17 volts appears on the meter (Fig. 11). When the switch is released the pot, now set correctly to 0.5317, is reconnected into the circuit.

The computer symbol for a potentiometer is simply a circle, as shown in Fig. 9.

Reference Voltage

At various points in a problem, constant voltages of different values are required. These are supplied from highly stabilized circuits which provide d-c voltages of +100 and —100 volts. These "reference" voltages are made available on the patch panel. To produce constant voltages other than 100 volts, the pots are used in conjunction with the reference voltage. For example, if the pot in Fig. 11 is set to 0.5317, and connected to the —100-volt reference, the output is —53.17 volts.

The Integrator

The basic equations of physics and engineering are differential equations—that is, variables expressed in terms of their derivatives, or rates of change. The following is a simple ordinary differential equation:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0 \quad (4)$$

If one has d^2x/dt^2, dx/dt can be obtained by performing one integration. If one has dx/dt, one can obtain x by performing an integration of dx/dt. Mathematically, this is written

$$\int \frac{d^2x}{dt^2}\,dt = \frac{dx}{dt} + C_1; \quad \int \frac{dx}{dt}\,dt = x + C_2$$

The device that performs integration in the analog computer is called an integrator (Fig. 12).

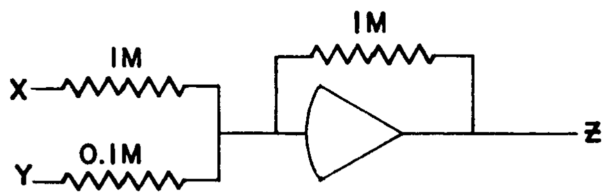
The integrator itself is simply an amplifier with a capacitor as its feedback* (Fig. 13). Comparing Fig. 13 with Fig. 5, one can see that the integrator and summer are identical except for the feedback; the integrator uses a capacitor, the summer a resistor.

If several input resistors are used, the amplifier integrates and adds simultaneously—that is, the output is the negative of the integral of the sum of the inputs. If the inputs are $x_1, x_2 \ldots . x_n$, and the corresponding input resistors are $R_1, R_2, \ldots . R_n$, then the output is:

$$Z = -\int_0^t \left[\frac{x_1}{R_1C} + \frac{x_2}{R_2C} + \ldots . \frac{x_n}{R_nC}\right] dt$$

*Making the same approximations as before (ignoring the input current to the amplifier and dropping all terms involving the factor 1/A), we obtain the expression for the output of an amplifier with a feedback capacitor (Fig. 13).

$$Z = -\frac{1}{RC}\int x\,dt$$



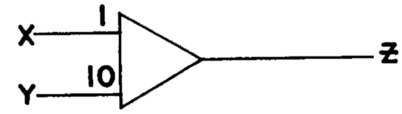


FIG. 5. SUMMING amplifier (left) and its computer symbol (top). Output $Z = -(X + 10Y)$

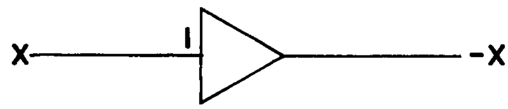


FIG. 6. WITH only one input as shown an amplifier is simply an inverter.

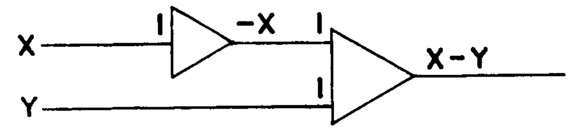


FIG. 7. AMPLIFIERS used for subtraction. Here two amplifiers are used to obtain $X - Y$.

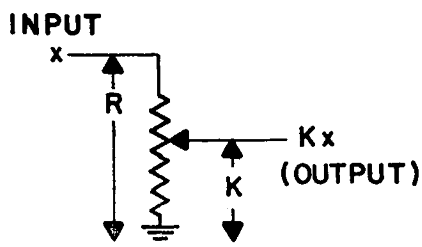


FIG. 8. A POTENTIOMETER is simply an attenuator.

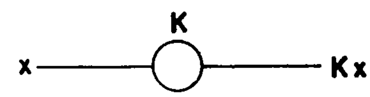


FIG. 9. SYMBOL for a potentiometer is a circle.

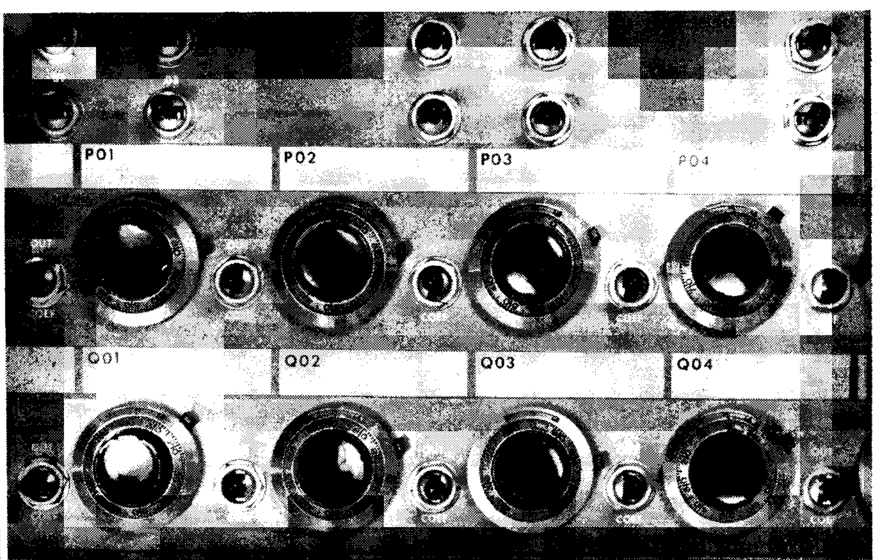


FIG. 10. POTENTIOMETERS on the 231R. Note that each "pot" is equipped with a switch for use in setting to high accuracy.

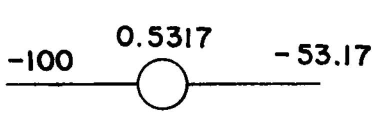


FIG. 11. PRODUCING a constant (— 53.17 volts) by using the reference voltage and a potentiometer.

FIG. 12. INTEGRATOR symbol is a rectangle against a triangle.

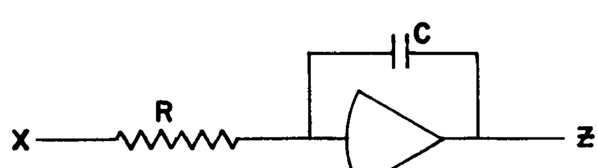


FIG. 13. INTEGRATOR is simply an amplifier with a feedback capacitor.

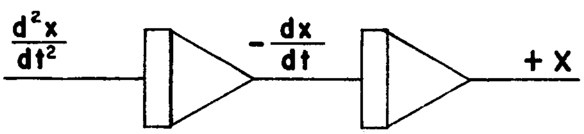


FIG. 14. **EACH INTEGRATION** reduces the order of the input derivative.

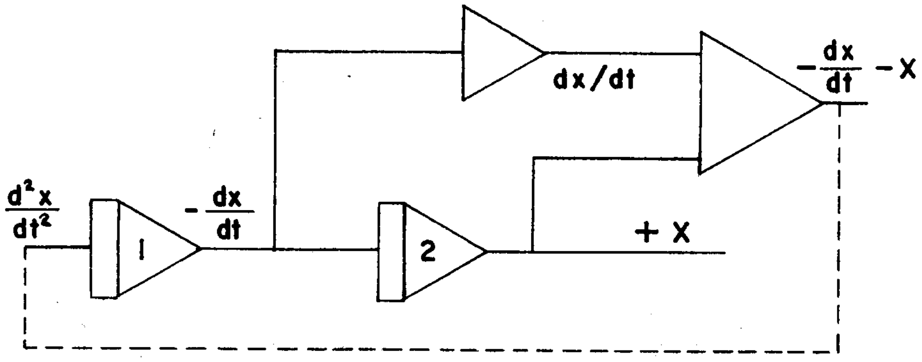


FIG. 15. **SUMMING** the dx/dt and x terms provides the right side of equation (5); thus equation (5) is satisfied by joining the output and input as shown by the dotted line.

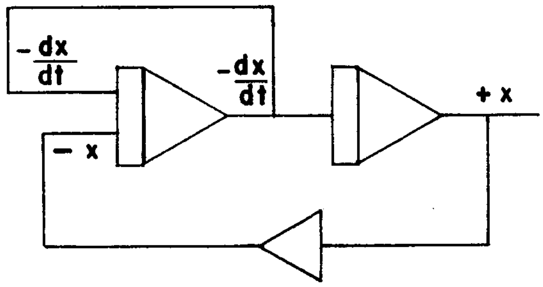


FIG. 16. **INTEGRATOR** can sum as well as integrate.

Simple Programming

The output of an integrator is always one order lower than the input. Conversely, the input of an integrator is always one order higher than the output. *This fact is the key to setting up and using the analog computer.*

The basic steps in setting up the differential equation of the analog computer are:

1. Start with the highest-order derivative.

2. Integrate the highest-order derivative to obtain the next lower-order derivative.

3. Repeat the process to obtain all derivatives of desired order.

4. Multiply each term by the desired constants, as given by the equation.

5. Add the terms as given by the equation.

6. Satisfy the equation by closing the loop with the proper terms.

Equation (4) will be used as an example. The first step is to rewrite the equation with the highest-order derivative on the left side:

$$\frac{d^2x}{dt^2} = -\frac{dx}{dt} - x \qquad (5)$$

This equation says that the highest-order derivative must equal the sum of the two terms on the right-hand side.

If we have a second-order derivative, we can obtain the first-order derivative by one integration, and then obtain x by a second integration, as shown in Fig. 14.

We wish to add dx/dt and x, but with the same sign. Thus we invert the term (dx/dt) and add it to x by using an adder, as shown in Fig. 15. The output of the summer is now the desired sum on the right side of equation (5), with the proper signs.

Equation (5) also says that this sum *must* equal d^2x/dt^2. This is realized by connecting the output of the summer (that is, the sum of the two terms in equation 5) to the input of the first integrator (which was assumed to equal d^2x/dt^2 at the beginning of the problem). The circuit in Fig. 15 now satisfies equation (5), and all voltages inside the circuit must change only as demanded by equation (5).

Equation (5) might represent an actual physical situation such as a wheel motion. The wheel system is now simulated by the circuit in Fig. 15. The variable x (which would be wheel displacement) can be examined at the output of integrator 2; the term dx/dt (which would be the velocity of the wheel) can be examined at the output of integrator 1; the term d^2x/dt^2 (which would be the acceleration of the wheel) can be examined at the output of the summer or at the input of integrator 1. Note that the displacement, velocity and acceleration of the wheel all are simulated by voltages in the circuit of Fig. 15.

The circuit shown in Fig. 16 uses the summation provided by the integrator (at left) to eliminate the separate summer shown at far right in Fig. 15. As the sum of the inputs ($-dx/dt-x$) is equal to d^2x/dt^2, this circuit also satisfies equation (5). Figs. 15 and 16 both solve the same problem, but Fig. 16 uses one less element.

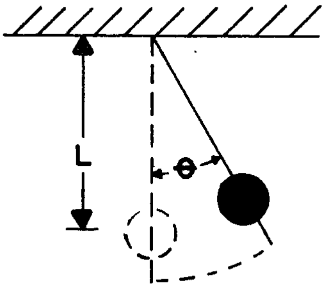


FIG. 17. SIMPLE pendulum.

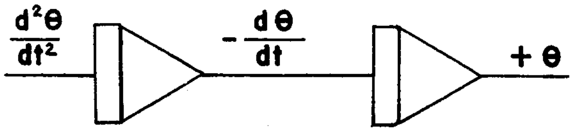


FIG. 18. IF $d^2\theta/dt^2$ is available, one integration gives $-d\theta/dt$; another integration gives $+\theta$.

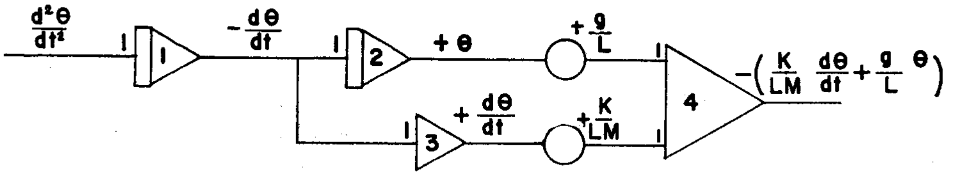


FIG. 19. USING POTS to multiply $d\theta/dt$ by k/L, and θ by g/L. Adding these two terms gives k/L $(d\theta/dt)$ + g/L (θ).

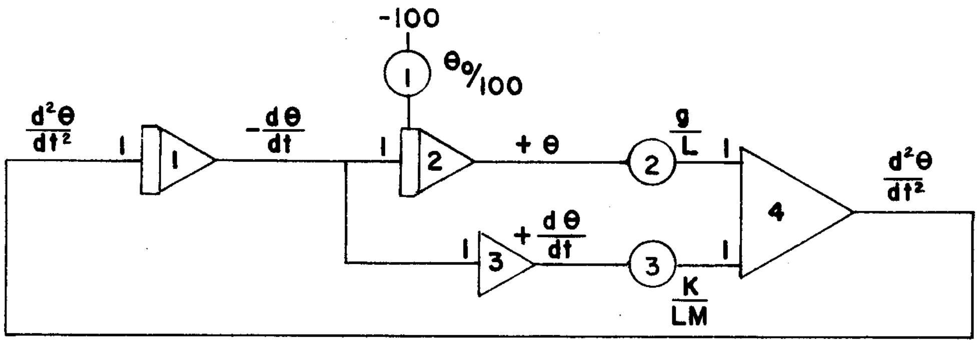


FIG. 20. SINCE output of summer-amplifier (4) is equal to the input of integrator (1), the circuit is completed as shown. Pot 1 is connected to the Initial Condition (IC) terminal of amplifier 2 to provide the proper initial condition θ_0.

Sample Problem

The three components described—summers, pots, and integrators—are sufficient to solve many basic differential equations that appear in physics and engineering. The simple pendulum problem is an example:

The pendulum bob shown in Fig. 17 is acted on by two forces along its path—gravity and friction.

1. The tangential force due to gravity = Mg sin θ; it depends on the mass M, gravity (g), and angular displacement (θ)

2. The air friction (damping) force = $K(d\theta/dt)$. Note that this force depends on velocity ($d\theta/dt$) and a friction constant K.

The equation of rotational motion is obtained by equating all *torques* about the pivot (including the rotational inertia term) to zero:

Torque 1 due to gravity = MgL ($\sin\theta$)
Torque 2 due to friction $K(d\theta/dt)$ = $KL(d\theta/dt)$.
Rotational inertia of mass M = ML^2
Rotational acceleration = $d^2\theta/dt^2$
Torque due to rotational acceleration = $(ML^2)(d^2\theta/dt)$

Hence

$$ML^2\left[\frac{d^2\theta}{dt^2}\right] + KL\left[\frac{d\theta}{dt}\right] + MgL\,(\sin\theta) = 0$$

The first step is to solve for the highest-order derivative:

$$\frac{d^2\theta}{dt^2} = -\frac{K}{LM}\left[\frac{d\theta}{dt}\right] - \frac{g}{L}\,(\sin\theta) \qquad (6)$$

To simplify matters, we can assume that θ is always sufficiently small (less than 15°) so that we can approximate sin θ by θ. This yields the equation:

$$\frac{d^2\theta}{dt^2} = -\frac{K}{LM}\left[\frac{d\theta}{dt}\right] - \left[\frac{g}{L}\right]\theta \qquad (7)$$

If we have a voltage proportional to $d^2\theta/dt^2$, then we can integrate once to find $d\theta/dt$ and again to find θ, as in Fig. 18.

We can multiply $(d\theta/dt)$ by (K/LM) and θ by (g/L) as shown in Fig. 19. If we add these two terms in a summer, we obtain the right-hand side of equation (7), as shown in Fig. 19. This sum must be $d^2\theta/dt^2$ according to equation (7). (Note the sign reversal every time an amplifier is used.)

Since amplifier 4 has the output $d^2\theta/dt^2$, we may use it for the input to amplifier 1, as shown in Fig. 20. This completes the circuit and satisfies equation 7.

Let us consider the case where the bob is given an initial displacement (θ_0) of ¼ radian and has an initial velocity of zero.

Since θ does not start at zero, the "initial condition" is imposed on θ by charging the feedback capacitor of integrator 2, which develops θ. This initial condition θ_0 is fed into an IC terminal on amplifier 2. The voltage at the output of amplifier 2 (θ) now satisfies the differential equation and the initial conditions $\theta=\theta_0$ and $d\theta/dt = 0$. The voltage representing θ therefore will vary in the same manner as the angle θ in the original problem.

Servomultipliers

Equation 7 is so simple that an analytic solution is obtained easily and one normally would not use a computer to solve such an equation. In solving the pendulum problem, we made two assumptions—

(1) that the damping was proportional to velocity and (2) that θ was so small that sin θ could be replaced by θ without serious error. These assumptions had the effect of making the differential equation *linear*. Suppose, however, that the damping term is proportional to some power of the angular velocity, say $(d\theta/dt)^3$. Then equation 7 becomes

$$\frac{d^2\theta}{dt^2} = -\frac{K}{LM}\left[\frac{d\theta}{dt}\right]^3 - \frac{g}{L}(\theta) \qquad (8)$$

Note that we are still assuming that θ is small.

Equation 8 is a nonlinear differential equation. An analytic solution of (8) is difficult if not impossible. To solve it on the computer, we use a device that generates x^3 from x. This is done by using a device that multiplies variables so that x can be multiplied by x or y or x^2 or any desired variable.

As we have seen (Fig. 8), a variable voltage (x) can be multiplied by a *constant coefficient* (K) by using a potentiometer. To multiply one *varying quantity* by another, we can use a pot whose slider is automatically positioned by an electric motor to follow a second variable. This type of multiplier is known as a *servomultiplier.*

If we have two varying voltages (x and y in Fig. 21) and wish to produce a voltage proportional to xy, we use one potentiometer whose input is y and whose sliding setting is proportional to x. The output would by proportional to xy.

In practice, this is achieved by using two potentiometers which are ganged— i. e., mounted on a common shaft—and using a motor to turn this shaft (Fig. 21). One of the pots is connected to a reference voltage (100 volts). Its arm is used as one of the inputs to a comparison network whose other input is x.

Pot 1 in Fig 21 has a wiper (arm) that is positioned by the motor, which receives a signal only when voltage on the wiper is not exactly equal to x. Thus the wipers on both pots follow every change in x, and the output from pot 2 is proportional to xy. As shown in Fig. 21, the actual output is xy/100.*

There is no reason to restrict ourselves to just two ganged pots; the servomultipliers in the 231R use six, labeled A through F (Fig. 22). The "F" potentiometer is connected to the reference voltage, and plays the role of pot 1 in Fig. 21. If x is the variable positioning the servo, then all six pots have the arm position x/100, and pots A through E all are available for multiplication by x. This arrangement enables one to generate terms such as xy, xz, . . . etc, with only one servomultiplier. This feature is especially useful in generating polynomials. In particular, it allows us to generate the cubic term in Equation 8 with just one servo, as shown in Fig. 23.

This scheme works only if x is a positive voltage because only positive voltages are available along the reference pot winding. If it is to be both positive and negative, the arrangement in Fig. 24 is used. There now is no restriction on the sign of the voltage x. Note also that pot 2 now has both +y and —y voltages at its terminals. The variable —y is obtained simply by using an inverter, as shown in Fig. 25, which also shows the symbol for a servomultiplier. Note in the symbol that one input goes to a box marked SM (number); the pot that is ganged to the motor-driven pot is identified by having the same number followed by a letter—(1A), (1B), etc. All pots that are ganged to one servomultiplier have the same number. Thus pot 4C means the third pot (C) ganged to the number 4 servomultiplier.

Fig. 26 shows how nonlinear equation (8) is implemented and solved. Note that the computer solution for equation (8) is scarcely more complicated then that of equation (7)—yet the analytic solution for equation (8) is much more difficult than that of (7).

*The output from pot 2 is xy/100, and not xy as desired, because of the 100-volt supply used with pot 1. If a 1-volt reference had been used for pot 1, the output from pot 2 would be exactly xy. When a 100-v supply is used on pot 1, then x (output from pot 1) = 100k, where k is the fraction of pot 1. Hence k (fraction of the pot 1) is x/100. Since pot 2 is ganged to pot 1, its setting is also x/100, and its output must be xy/100.

The factor of 100 also can be appreciated by noting that x and y each can vary from 0 to 100 volts. Hence the maximum output of the product xy should be 100 x 100, or 10,000 volts. However, the maximum voltage from pot 2 is 100 volts. Hence the output from pot 2 must be xy/100. Note that if pot 1 had had a 1-volt supply, the output from pot 2 would be exactly xy; if pot 1 had had a 10-volt supply, the output from pot 2 would be xy/10, etc.

The Quarter-Square Multiplier

Another-widely used multiplying device is the *quarter-square multiplier.* This device uses the algebraic identity:

$$xy = \tfrac{1}{4}[(x + y)^2 - (x - y)^2]^*$$

To generate xy we need only perform addition (x + y), subtraction (x — y) and squaring of the two terms. Squaring turns out to be much simpler than multiplication because it involves only one variable. (The means of generating the square and other functions of a single variable will be discussed later.) As the quarter-square multiplier is all electronic, it has a much higher frequency response than the servomultiplier.

Function Generators

Equation (8) was simplified by the replacement of sin θ by θ. If we are to remove this restriction, we need a device which will accept an input voltage proportional to θ and produce a voltage proportional to sin θ. This can be done by a *function generator.*

Many types of function generators are available, but the variable *diode function generator* (DFG) is typical. It employs a network of resistors and diodes to approximate the given function by use of straight-line segments, as shown in Fig. 27. The slope and breakpoint of each segment can be individually adjusted to allow a best fit to the curve (function).

A *fixed* DFG consists of a printed-circuit card with specific components chosen to produce the desired function. Commonly used functions such as log x, x^2, x^4, tan x, sin x, etc. are available. The symbol for either the fixed or variable type of DFG is given in Fig. 28.

$$\begin{aligned}{}^*xy &= \tfrac{1}{4}[(x + y)^2 - (x - y)^2]\\ &= \tfrac{1}{4}(x^2 + 2xy + y^2 - x^2 + 2xy - y^2)\\ &= \tfrac{1}{4}(4xy) = xy\end{aligned}$$

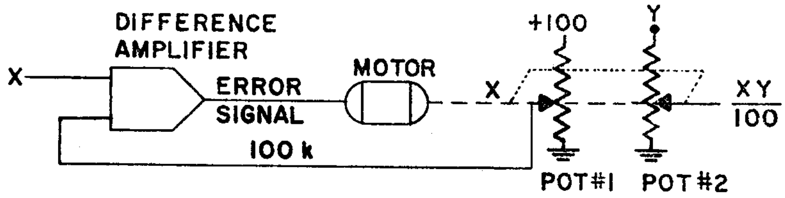


FIG. 21. PRINCIPLE of multiplication of two variables. If top of pot is a variable Y, and setting is a variable X, then output is proportional to XY.

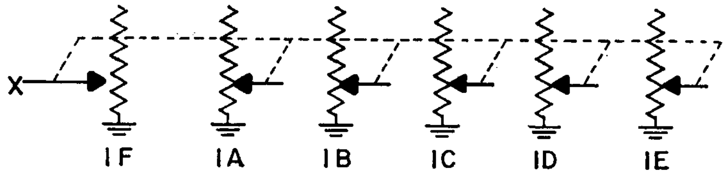


FIG. 22. SERVOMULTIPLIFER positions the taps on six ganged pots —one follower pot (F) and five multiplying pots (A-E).

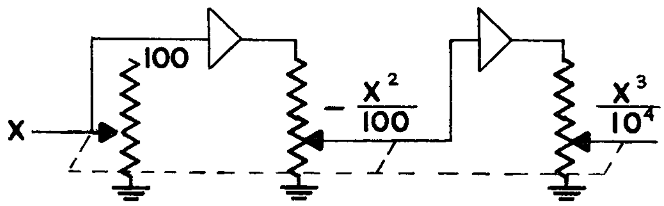


FIG. 23. GENERATING the cube of a variable with only one servomultiplier.

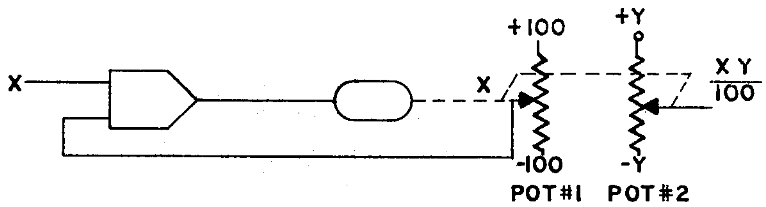


FIG. 24. SERVOMULTIPLIER in which variable X can have any value between —100 and +100 volts. Note that both references (— 100 and + 100 v) are needed, and also +X and —Y.

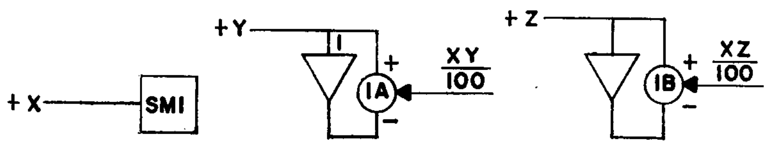


FIG. 25. SYMBOLS for a servomultiplier. Five pots (A-E) are ganged to the motor-driven pot (F) to permit X to be multiplied by five variables simultaneously.

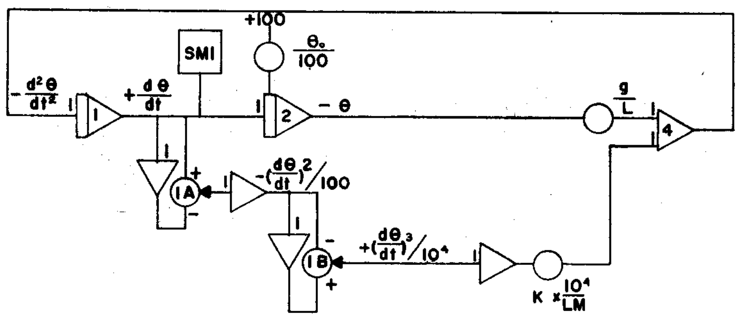


FIG. 26. CIRCUIT for solving the pendulum problem with cubic damping term. Note that we can perform two multiplications by the factor $d\theta/dt$ to produce $(d\theta/dt)^3$ with only one servomultiplier.

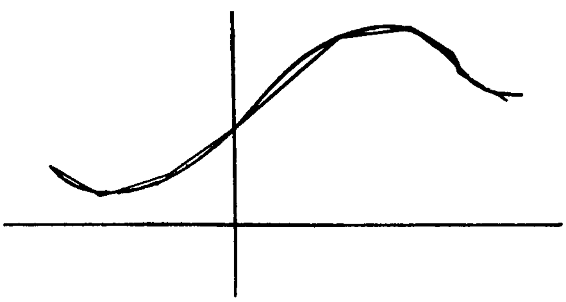


FIG. 27. APPROXIMATING a smooth curve by straight-line segments.

FIG. 28. SYMBOL for a diode function generator.

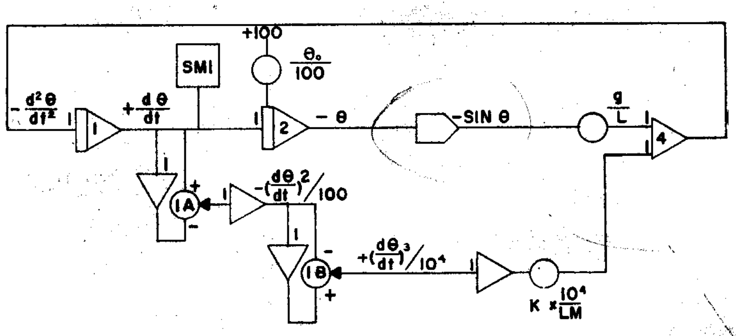


FIG. 29. **CIRCUIT** for solving the equation $d^2\theta/dt^2 = -(K/LM)(d\theta/dt)^3 - g/L(\sin\theta)$. Note that the DFG is the only difference between this circuit and that of Fig. 26.

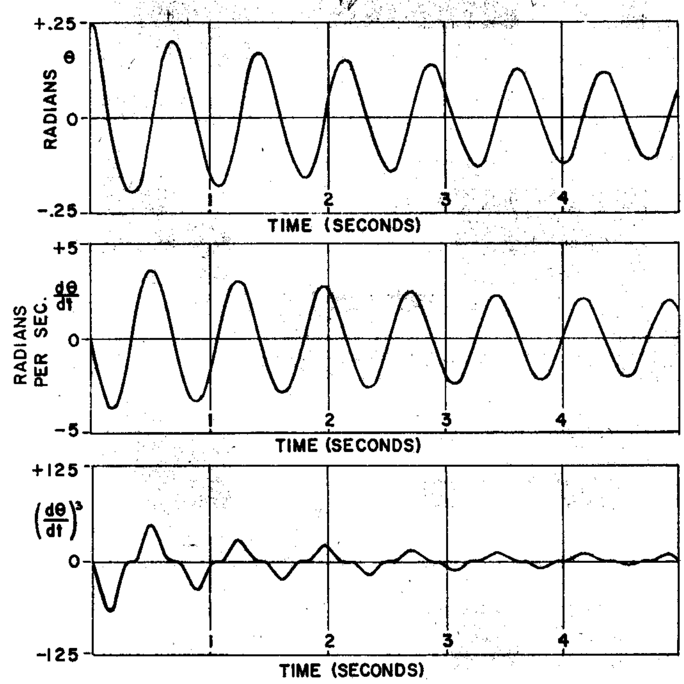


FIG. 30. **SOLUTION** to $d^2\theta/dt^2 = -2(d\theta/dt)^3 - 12\theta$, with initial conditions: $\theta = 0.25$ radian and $d\theta/dt = 0$.

The accuracy (closeness of fit to original function) of the DFG depends primarily on the number of segments used. Ten- and twenty-segment DFG's are available on the 231-R; these are sufficient for most practical problems.

The pendulum problem can now be solved without any simplifying assumptions because we can use a DFG set up to give an output $\sin\theta$ for an input θ. The computer program for the nonlinear pendulum is given in Fig. 29. (Other techniques can be employed to give a better approximation to $\sin\theta$, but this simple application illustrates the basic principles.)

Recording Devices

We have seen how to use a computer to obtain time-varying *voltages* that represent the solution to a mechanical problem. To observe and interpret these results we need measuring and recording devices.

One such unit, the *strip-chart recorder* employs a roll of paper drawn at a constant speed past a pen which is deflected proportional to the input voltage. The pen thus draws a graph of the computer voltage as a function of time. Typical models have parallel channels on a single strip of paper, allowing the operator to record several variables simultaneously. Fig. 30 shows the solution of equation (8) recorded on a strip-chart recorder. The three channels shown record the displacement (θ), velocity ($d\theta/dt$) and the velocity cubed term $(d\theta/dt)^3$. In a more complex problem, more channels might be needed.

Another recording device, the *X-Y plotter,* uses a moving pen and stationary paper. This device has two voltage inputs, enabling one to plot one varying voltage against another. The pen moves back and forth along a straight arm, its deflection being proportional to one voltage. The arm itself moves in a perpendicular direction positioned by the second voltage. On 10″ x 15″ paper ruled in 1/10″ squares; the resulting graph can be read with an accuracy of about 1 part in 1000. Plotters for 11″ x 17″ and 30″ x 30″ graphs are popular; even larger sizes are available.

Problem Checking

Many tests have been devised to assure the operator that the problem has been programmed correctly and that all components are functioning properly. One of these, the *static test,* is a check on both the program and the equipment.

The standard procedure consists of choosing an arbitrary set of initial conditions—that is, a set of values for the unknown variable and all its derivatives appearing in the problem, except the highest derivative. In the pendulum problem, this means chosing initial values for θ and $d\theta/dt$. The highest derivative (in this case $d^2\theta/dt^2$) then can be computed, and from this the voltage representing $d^2\theta/dt^2$ can be calculated. For example, suppose we are solving equation (7) for the case, $k/LM = 0.5$ and $g/L = 0.8$. If we assume $\theta=0.25$ radian (about 15°) and $d\theta/dt = 1$ rad/sec as initial conditions, we may solve equation (7) for the initial value of $d^2\theta/dt^2$:

$$\frac{d^2\theta}{dt^2} = -0.5(1) - 0.8(0.25) = -0.7 \text{ rad/sec}^2$$

The assigned initial values for $d\theta/dt$ and θ correspond on the computer diagram (Fig. 29) to scaled initial voltages of the outputs of integrators 1 and 2. Similarly, the calculated value for $d^2\theta/dt^2$ corresponds to a voltage output of the summing amplifier (No. 4). Since we know the initial outputs of the integrators, we can calculate the output of amplifier 4 from the circuit diagram. Comparing this scaled voltage to the calculated value for $d^2\theta/dt^2$, we have a check on the correctness of the diagram.

The final step in the static-check procedure is taken after the problem is put on the computer, as follows:

Voltages proportional to the initial conditions (in this case $d\theta/dt = 1$ and $\theta = 0.25$) are established at the "Initial Condition" terminals of amplifiers 1 and 2, and the output of amplifier 4 is measured and compared with the calculated value. This check verifies the patching and the functioning of the computer components. Most errors, human or mechanical, are discovered and corrected during the static-test procedure.

Automatic Setup

As computers increase in size to allow solution of more complex problems, there is a trend toward automatic devices to reduce set-up time and minimize the possibility of human error. The ADIOS (Automatic Digital Input-Output System) together with DAS (Digital Attenuator System) available on the 231-R is an example. Consisting of an electric typewriter, keyboard, paper-tape punch and tape reader, together with the necessary relays and switches, the ADIOS enables the operator to set up potentiometers and check programming from either the keyboard or tape. To set a potentiometer for 0.2317, the operator simply punches out on the keyboard an address code for the potentiometer desired and the digits 2317. A servomechanism in the computer automatically sets the pot to the desired value. Both potentiometers and function generators can be set up in this fashion, resulting in a much faster computer set-up and problem check. The combination of a paper tape and patch panel allows for complete problem storage.

Repetitive Operation

When the variation of many parameters is required in a design study, full advantage may be taken of the analog's high speed by the addition of *repetitive operation* (*rep-op*), a feature available on many computers. In repetitive operation, the speed of solution is increased, generally, by changing the value of the feedback capacitors on all integrators, and using high-speed relays to repeatedly reset the initial conditions on these integrators, allow the solution to run, and then reset again. The resultant voltage output (viewed on an oscilloscope) appears as a continuous curve—a graph of the solution *versus* time. When the pots (representing parameters) are varied, the result is an almost instantaneous change in the output graph. This visual display enables the operator to find the optimum parameters quickly.

Analog Versus Digital

Invariably the question arises—Which is better, an analog or a digital computer? The answer is that neither is always better; the best choice depends on the particular problem to be solved. While there are many considerations, in general, the principal advantage of the analog computer is speed, while the advantage of the digital computer is accuracy.

The speed advantage of the analog computer arises from the fact that while the digital computer performs its calculations with discrete numbers in sequence., the analog computer has all components (summers, integrators, etc.) performing their operations simultaneously and continuously. Thus, the digital computer must wait until each calculation is completed before moving on to the next. The more complex the problem, the more calculations are required and the longer the time required for solutions. The time required for solutions on the analog is virtually independent of the complexity of the problem. A typical analog computer solution takes from 10 to 60 seconds, with 20 seconds a good average figure. One large analog computing laboratory which has acquired a digital computer for checking purposes reports an analog speed advantage of over 100 to 1 in some problems.

This speed advantage is especially important if a problem is to be solved for many parameter variations. The stiffness of a spring, the wing span of an airplane, and the gain of a pneumatic controller are all examples of parameters that can be set on pots and varied between runs, enabling the operator to judge quickly the effect of any change in parameters. In a design problem with many adjustable parameters, this speed advantage usually enables the optimum design to be reached more quickly than by digital methods.

The accuracy of an analog computer is limited by the accuracy of the electrical components, and three significant figures is the limit of accuracy presently attainable in medium-sized problems. On a digital computer, the accuracy is determined chiefly by the number of decimal places (actually binary places) retained in calculation and, with sufficient equipment and time, high accuracy can be achieved. If accuracies higher than about 1% are necessary, a digital solution is indicated; however, most engineering problems do not require this high accuracy because in many cases the input data are not this accurate. In these cases the high speed of an analog, and the fact that it produces easily interpreted continuous curves rather than discrete points, make an analog solution preferable.